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HJB-inequalities in estimating reachable sets of a control system under uncertainty. (English)

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A class of nonlinear control system is considered in the presence of unknown but bounded uncertainties. The aim is to estimate the corresponding reachable sets. The proposed approach is based on the techniques of the developed ellipsoidal calculus and also uses some ideas related to the Hamilton-Jacobi-Bellman equation. The main result provides (under suitable conditions) differential equations that describe the dynamics of the external ellipsoidal estimates of the reachable sets. Moreover, these estimates are optimal in the sense that they can not be reduced without violating the requirement to be outer ellipsoidal estimates of the reachable sets. Numerical simulations are presented to illustrate the proposed approach.

Reviewer: [Mikhail I. Krastanov \(Sofia\)](#)

MSC:

[93B03](#) Attainable sets, reachability

[93C10](#) Nonlinear systems in control theory

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[nonlinear control system](#); [reachable set](#); [unknown but bounded uncertainties](#); [external ellipsoidal estimate](#)

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