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Bundles with non-multiplicative \( \hat{A} \)-genus and spaces of metrics with lower curvature bounds.
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Let \( M \) be a smooth closed \( d \)-dimensional manifold. Then \( \mathcal{R}(M) \) denotes the space of all metrics on \( M \) with the \( C^\infty \)-topology. Let \( \mathcal{R}_{\text{scal}>0}(M) \) denote the subspace of \( \mathcal{R}(M) \) of all metrics with positive scalar curvature. In their paper, the authors spot elements of infinite order of homotopy groups for a large class of manifolds. Let \( \mathcal{F}(M) \subset \mathcal{R}(M) \) be a \( \text{Diff}(M) \)-invariant subset that admits a \( \text{Diff}(M) \)-equivariant continuous map \( \iota_F : \mathcal{F}(M) \to \mathcal{R}_{\text{scal}>0}(M) \) (for example, this includes \( \mathcal{R}_{\text{Ric}>0}(M) \)). Let \( W^d_g = (S^n \times S^n)^\#g \) and \( W^{2n+1}_g = (S^n \times S^{n+1})^\#g \). The authors define the genus of \( M \) as the largest number \( g \) such that there is a manifold \( N \) with \( M \cong N \# W^d_g \).

Authors’ main result (Theorem A) states that if \( M \) is a Spin-manifold of genus at least 1 and \( d \geq 10, d \neq 13 \), and if \( \mathcal{F}(M) \neq \emptyset \), then either \( \pi_1(\mathcal{F}(M)) \) is infinite or \( \pi_j(\mathcal{F}(M)) \otimes \mathbb{Q} \neq 0 \) for some \( 2 \leq j \leq 9 \).

For \( d = 6 \mod 8 \), the authors prove a stronger result (Theorem B): either the map \( \iota_F : \mathcal{F}(M) \to \mathcal{R}_{\text{scal}>0}(M) \) collapses infinitely many path components to one or \( H_1(\mathcal{F}(M); \mathbb{Q}) \neq 0 \).

The authors conclude the paper with examples of manifolds \( M \) for which the image of the map on homotopy groups induced by the orbit map \( \text{Diff}(M, D) \to \mathcal{R}_{\text{scal}>0}(M) \) contains elements of infinite order, where \( \text{Diff}(M, D) \) is the topological group of diffeomorphisms of \( M \) fixing an embedded disk \( D \subset M \) of codimension zero.

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MSC:

- 58D17 Manifolds of metrics (especially Riemannian)
- 53C21 Methods of global Riemannian geometry, including PDE methods; curvature restrictions
- 55P47 Infinite loop spaces
- 55R35 Classifying spaces of groups and \( H \)-spaces in algebraic topology
- 57R22 Topology of vector bundles and fiber bundles
- 58D05 Groups of diffeomorphisms and homeomorphisms as manifolds

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