Let $M$ be a (connected, oriented, smooth, but not necessarily closed) manifold of dimension $d \geq 2$. Consider a fixed embedding of $m \geq 1$ disjoint closed $d$-dimensional discs, denoted $\bigsqcup m D^d$, into $M$. Moreover, fix a finite set $x \subset \partial M \setminus \bigsqcup m D^d$ of cardinality $k$. Let $\text{Diff}^{+,k}_m(M)$ denote the group of orientation-preserving diffeomorphisms of $M$ that are

1. the identity in a neighborhood of the boundary $\partial M$,
2. permute the components of $\bigsqcup m D^d$ in a standard way, i.e., a fixed parametrization of each of the discs is preserved, and
3. fix the set $x$ set-wise (but not necessarily point-wise).

Then any such diffeomorphism permutes the points and discs, and gives an underlying diffeomorphism of $M$ by the forgetful map. This yields a homomorphism of topological groups

$$\text{Diff}^{+,k}_m(M) \to \text{Diff}^+(M) \times \Sigma_m \times (\Sigma_k \wr \text{GL}^+_d(\mathbb{R})),$$

where $\text{Diff}^+(M)$ denotes the topological group of orientation-preserving diffeomorphisms of $M$ that are the identity near the boundary, and $\Sigma_n$ means the symmetric group on $n$ letters. (The map to the wreath product $\Sigma_k \wr \text{GL}^+_d(\mathbb{R})$ also considers the differentials of the diffeomorphisms at the points in $x$.)

C.-F. Bödigheimer and U. Tillmann showed [Prog. Math. 196, 47–57 (2001; Zbl 0992.57014)] that this map induces a homology isomorphism on classifying spaces in case $d = 2$, i.e., $M$ is a surface, in a range of degrees growing with the genus of $M$.

The main purpose of this paper is to extend this result to simply-connected manifolds of higher even dimensions and also allow for other tangential structures than orientation. The central ingredient in the proof is the higher-dimensional analogue of the Madsen-Weiss theorem established by S. Galatius and O. Randal-Williams [in: Handbook of homotopy theory. Boca Raton, FL: CRC Press. 443–485 (2020; Zbl 1476.57058)].

A corollary is the computation of the stable cohomology of $\text{Diff}^{+,k}_m(W_{g,1})$, where $W_{g,1} = (\#^g S^n \times S^n) \setminus D^{2n}$ for $2n \geq 6$. This manifold can be seen as the $2n$-dimensional analogue of a surface of genus $g$ with one boundary component.

Reviewer: Jens Reinhold (Münster)

MSC:
- 55R40 Homology of classifying spaces and characteristic classes in algebraic topology
- 55N99 Homology and cohomology theories in algebraic topology
- 55R10 Fiber bundles in algebraic topology
- 57R15 Specialized structures on manifolds (spin manifolds, framed manifolds, etc.)

Keywords:
- diffeomorphism groups; moduli spaces of manifolds; homological stability; classifying spaces; cohomology of groups

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