On the optimality of pseudo-polynomial algorithms for integer programming.

Summary: In the classic Integer Programming Feasibility (IPF) problem, the objective is to decide whether, for a given $m \times n$ matrix $A$ and an $m$-vector $b = (b_1, \ldots, b_m)$, there is a non-negative integer $n$-vector $x$ such that $Ax = b$. Solving (IPF) is an important step in numerous algorithms and it is important to obtain an understanding of the precise complexity of this problem as a function of natural parameters of the input.

The classic pseudo-polynomial time algorithm of C. H. Papadimitriou [J. Assoc. Comput. Mach. 28, 765–768 (1981; Zbl 0468.68050)] for instances of (IPF) with a constant number of constraints was only recently improved upon by F. Eisenbrand and R. Weismantel [SODA 2018, 808–816 (2018; Zbl 1410.90128); ACM Trans. Algorithms 16, No. 1, Article No. 5, 14 p. (2020; Zbl 1454.90029)] and K. Jansen and L. Rohwedder [LIPIcs – Leibniz Int. Proc. Inform. 124, Article 43, 17 p. (2019; Zbl 1502.68138)]. Jansen and Rohwedder designed an algorithm for (IPF) with running time $O(m\Delta) m \log(\Delta) \log(\Delta + \|b\|_\infty) + O(mn)$. Here, $\Delta$ is an upper bound on the absolute values of the entries of $A$. We continue this line of work and show that under the Exponential Time Hypothesis (ETH), the algorithm of Jansen and Rohwedder is nearly optimal, by proving a lower bound of $n^{o(\frac{m}{\log m})} \cdot \|b\|^{o(m)}$. We also prove that assuming ETH, (IPF) cannot be solved in time $f(m) \cdot (n \cdot \|b\|_\infty)^{o(\frac{m}{\log m})}$ for any computable function $f$. This motivates us to pick up the line of research initiated by W. H. Cunningham and J. Geelen [Lect. Notes Comput. Sci. 4513, 158–166 (2007; Zbl 1136.90403)] who studied the complexity of solving (IPF) with non-negative matrices in which the number of constraints may be unbounded, but the branch-width of the column-matroid corresponding to the constraint matrix is a constant. We prove a lower bound on the complexity of solving (IPF) for such instances and obtain optimal results with respect to a closely related parameter, path-width. Specifically, we prove matching upper and lower bounds for (IPF) when the path-width of the corresponding column-matroid is a constant.

MSC:
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90C10 Integer programming

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References: