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Suslin homology via cycles with modulus and applications. (English)

One of primary motivations in the theory of cycles with modulus arises from an interest in offering some descriptions of certain relative $K$-theory and $K$-theory of singular schemes. These groups $\text{CH}_p(X|D, n)$ are defined for pairs $(X, D)$ consisting of a scheme $X$ and an effective Cartier divisor $D$ on $X$.

Recent years, there have been numerous activities and results in understanding these groups. Even when $n = 0$, some of these are nontrivial. For instance, the theorem of M. Kerz and S. Saito [Duke Math. J. 165, No. 15, 2811–2897 (2016; Zbl 1401.14148)] offers a description of an abelianized fundamental group of a variety in terms of algebraic cycles, which is an advance in class field theory of varieties.

The present article under review is, in a sense, a sequel to an earlier article by the authors [J. Éc. Polytech., Math. 9, 281–325 (2022; Zbl 1486.14013)], in that this paper also concentrates on the study of the natural surjection

$$\phi_{X|D} : \text{CH}_0(X|D) \to H_0^{\text{Sus}}(U),$$

from the Chow group of 0-cycles associated to the modulus pair $(X, D)$ to the 0-th Suslin homology of $U = X \setminus D$, where $X$ is smooth projective over a field $k$ and $D$ is assumed to be reduced. Previously in [loc. cit.], this map $\phi_{X|D}$ was proven to be an isomorphism when $k$ is algebraically closed field of positive characteristic. This result is included for completeness as part of the main theorem of the present article, more precisely as Theorem 1.1-(3).

The new contributions made in this article offer three additional cases when the map $\phi_{X|D}$ are isomorphisms, specifically as Theorem 1.1-(1), (2), and (4). The authors prove that, when

- $D$ is a simple normal crossing divisor, or
- $k$ is perfect and $\dim (X) \leq 2$, with $D$ semi-normal, or
- $k \subset \mathbb{Q}$

the map $\phi_{X|D}$ is an isomorphism.

While this theorem might appear to some people as a basic extension of a pre-existing result, it does offer a few strong applications. Some of them are simpler arguments for known nontrivial results (e.g., the class field theory of Kerz-Saito), while some of them are genuine generalizations of certain conditionally known results (e.g., an analogue of Roitman torsion theorem for Suslin homology).

An important tool in proving the main theorem comes from the fundamental split short exact sequence (recalled as Theorem 3.2)

$$0 \to \text{CH}_0(X|D) \xrightarrow{p_*} \text{CH}_0^{l.c.i.}(S_X) \xrightarrow{\iota_*} \text{CH}_0^F(X) \to 0,$$

where $X$ is regular quasi-projective, $\text{CH}_0^{l.c.i.}(S_X)$ is the l.c.i. version of the Levine-Weibel Chow group of the double $S_X = X \bigsqcup_D X$ (recalled in the article), and $\text{CH}_0^F(X)$ is the classical Chow group. A key technical step is to show that the map $p_*$ factors via the Suslin homology $H_0^{\text{Sus}}(U)$, where $U = X \setminus D$, which is done in Lemma 3.3. The other important ingredient is a comparison with the homotopy $K$-theory $KH$ under the assumptions of simple normal crossing, etc.

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MSC:

- 14C25 Algebraic cycles
- 14F42 Motivic cohomology; motivic homotopy theory
- 19E15 Algebraic cycles and motivic cohomology ($K$-theoretic aspects)
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References:


